Question 1

Recall the marble drawing example from class. There are 4 marbles in a bag, each marble is either Blue/White. We picked out the sequence of Blue, White, Blue marbles, placing each back in the bag every time.

Here is the plausibility of the bag containing different numbers of blue marbles after we observe the data sequence B, W, B:

![Bar chart showing the plausibility of the bag containing different numbers of blue marbles after observing the sequence B, W, B.](image)

a) Compute a similar count distribution if we instead observed the sequence White White. How many blue marbles is the bag most likely to contain?
b) Compute a similar count distribution if we instead observed the sequence *White Blue Blue*. *Hint: Treat your answer from a) as the prior distribution.*

c) Convert your answer from b) to a probability distribution.

**Question 2**

a) Give an example covariance matrix of multivariate Gaussian distribution that is oriented such that the line $y = -x$ would mostly pass through its main axis of variation.

b) Is it possible to have the following covariance matrix? Why or why not?

$$
\Sigma = \begin{pmatrix}
2 & 1 \\
-1 & 2
\end{pmatrix}
$$
Question 3

Suppose that I have globes for Uranus and Earth. We know that the surface of Earth is 70% water and Uranus is 80% land (assume here that a planet can EITHER be covered by water or land). I tossed one of the globes without showing you which one and told you that the trial ended in a Land observation (either globe was equally likely to be tossed). Show that the probability that I tossed the Earth globe after hearing the Land observation is $\approx 0.27$.

Question 4

Let’s say that there are two species of gorilla, in equal populations within the same habitat. They seem indistinguishable: they eat the same food, look the same, and we don’t have a genetic test capable of differentiating them. Clever scientists have, however, discovered that they give birth to twins in different frequencies. Species 1 has twins 10% of the time (the rest of the time they birth a single infant), whereas Species 2 has twins 20% of the time (the rest of the time they birth a single infant). Assume that these numbers are well-established after many years of research.

a) Determine the probability that a gorilla is from Species 1, assuming that we have observed a birth and that it was twins.
b) How would your answer in a) change if we knew that the population of Species 2 was twice as numerous as Species 1?