Question 1: Naive Bayes

You are debugging a Naive Bayes model for spam email classification and would like to work out a simple test case to make sure your code is working correctly. You want to figure out by pen-and-paper whether your model should classify a test message containing the words you, won, lottery as Spam or Not Spam.

For your likelihood, you are using normalized counts from your training set (not a Gaussian distribution).

For your prior, you are using an internet spam word occurrence dataset and discovered that Not Spam emails are 3 times more likely as Spam emails.

Training counts

Assume that your entire training email data is summarized in the following table:

<table>
<thead>
<tr>
<th>Word</th>
<th>Count when Spam</th>
<th>Count when Not Spam</th>
</tr>
</thead>
<tbody>
<tr>
<td>you</td>
<td>40</td>
<td>10</td>
</tr>
<tr>
<td>won</td>
<td>60</td>
<td>40</td>
</tr>
<tr>
<td>lottery</td>
<td>100</td>
<td>50</td>
</tr>
</tbody>
</table>

Use Naive Bayes to determine whether the email (with you, won, lottery) is Spam or Not Spam.
Question 2: ROC Curve

A supervised learning model processed a medical database and produced the following confusion matrix:

<table>
<thead>
<tr>
<th></th>
<th>Predicted (+)</th>
<th>Predicted (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual (+)</td>
<td>70</td>
<td>30</td>
</tr>
<tr>
<td>Actual (-)</td>
<td>80</td>
<td>120</td>
</tr>
</tbody>
</table>

a) Define **true positive rate**.

b) Define **false positive rate**.

c) Define **precision** (positive predictive value).

c) In this example, the true positive rate is ____________, the false positive rate is ____________, and the precision is ____________.
d) In the graph above, place the point you computed from (c).

e) The above confusion matrix was generated with a binary decision threshold of $\gamma = 0.4$ (i.e. the point that you plotted in (d) corresponds to $\gamma = 0.4$).

i. For a different threshold value ($\gamma = 0.2$) estimate the false positive rate if

- Our classifier labels 90% of true positive cases as (+)
- Our classifier labels 120 cases that are actually (-) as (+)

ii. Place the $\gamma = 0.2$ point on the plot.

f) Sketch an ROC curve that includes the $\gamma = 0.4$ and $\gamma = 0.2$ points.

g) Use the "graph paper" to estimate the C-index/AUC value.

h) Is the AUC value "good", "mediocre", or "bad" compared to an all-negative classifier?