

Analysis of Algorithms
CS 375, Fall 2019

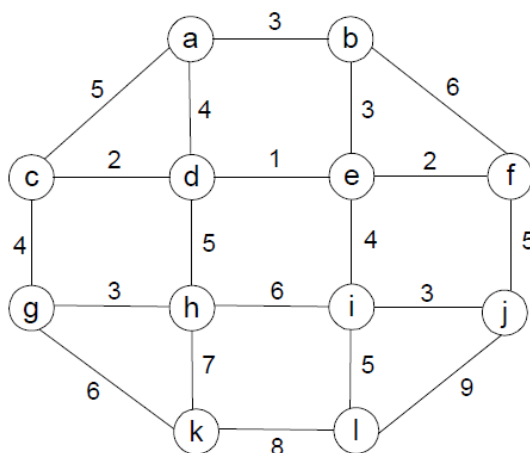
Homework 16

Due **AT THE BEGINNING OF CLASS** Wednesday, December 4

- From your textbook (CLRS), please read Chapter 34, pages 1048–1052, 1061–1070.
- When presenting an algorithm, describe it in English clearly, concisely, and unambiguously; pseudocode often helps clarify a presentation, but a pseudocode-only presentation is not acceptable. In general, unclear presentations may not receive full credit.
- *A general note:* When writing up your homework, please write neatly and **explain your answers clearly**, giving all details needed to make your answers easy to understand. Graders may not award credit to incomplete or illegible solutions. Clear communication *is* the point, on every assignment.

Exercises

1. Apply Kruskal's algorithm to the following graph, to find a minimum spanning tree. Show your work: To show that you understand Kruskal's algorithm, show the order in which each edge is added and, for each added edge, give a very brief (1 sentence or less is fine!) explanation of the reasons behind choosing that edge.



2. **Bridge Crossing Revisited!** Consider the generalization of the bridge crossing exercise earlier in the semester (HW1, exercise 6) in which there are $n > 1$ people whose bridge crossing times are $t_1, t_2, \dots, t_n$. All the other conditions of the problem are the same as before: at most two people at a time can cross the bridge (and they move with the speed of the slower of the two) and they must carry with them the only flashlight the group has.
 - (a) Design a greedy algorithm for getting the entire group across the bridge, with the goal of minimizing the total time for the group to cross. Give a very brief explanation of what makes it a greedy algorithm.

- (b) Show that the greedy algorithm does not always yield an optimal solution (i.e., a minimal crossing time) for every instance of this problem by giving a concrete counterexample **with the smallest number of people** for which it is not optimal.
3. Exercise 34-1 in CLRS (pages 1101–1102) defines the independent-set problem. As presented, it is an optimization problem. Formulate a related *independent-set decision problem* by giving the input / output specifications of the decision variant, and explain briefly (1 sentence or so should suffice) how it relates to the optimization problem.
4. The *bin-packing problem* is an optimization problem specified as follows:

Input Set $S = \{i_1, \dots, i_n\}$ of n items, where item i_z has associated rational-number size s_z ($0 < s_z \leq 1$)

Output Integer m , which is the smallest integer such that all items in S can fit into m bins of size 1.

For example, if S has three items all with size $\frac{1}{2}$, they could fit in 2 bins of size 1—two items in one bin, one item in the third. If S has five items of sizes $\frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}$, they could fit in three bins of size 1, but they could not fit in two bins of size 1.

Formulate a related *bin-packing decision problem* by giving the input / output specifications of the decision variant, and explain briefly (1 sentence or so should suffice) how it relates to the optimization problem.