

Analysis of Algorithms
CS 375, Fall 2019

Homework 9

Due **AT THE BEGINNING OF CLASS** Wednesday, October 16

- From your textbook (CLRS), please read all of Chapters 2 and 3, and pages 65–67 and 88–93 from Chapter 4. Although not all of it will show up on CS375 exercises, it is all good material to know.
- *A general note:* When writing up your homework, please write neatly and **explain your answers clearly**, giving all details needed to make your answers easy to understand. Graders may not award credit to incomplete or illegible solutions. Clear communication *is* the point, on every assignment.

Exercises

1. Using the *unwinding* method from class, solve the following recurrence relations and give the Θ class of the solution.
 - (a) $T(n) = T(n - 1) + 5$ for $n > 1$; $T(1) = 0$.
 - (b) $T(n) = 3T(n - 1)$ for $n > 1$; $T(1) = 4$.
 - (c) $T(n) = T(n - 1) + n$ for $n > 0$; $T(0) = 0$.
 - (d) $T(n) = T(n/2) + n$ for $n > 1$; $T(1) = 1$. (Assume n is of the form 2^k for this exercise.)
 - (e) $T(n) = T(n/3) + 1$ for $n > 1$; $T(1) = 1$. (Assume n is of the form 3^k for this exercise.)
2. Express the running time (as a function of n) of the following pseudocode algorithm using Θ -notation.

```
r = 0
i = n * n
while i > 1 do
    for j = 1 to i do
        r = r + 1
    i = i/2
```

In the above, $i/2$ is integer division. To simplify your analysis, you may assume that n is a power of 2.

Be sure to explain the details behind your answer: state what *barometer operation(s)* you counted to arrive at your answer; show all work that you did to get *an exact count* of those operations and arrive at a summation that expresses the running time; and briefly explain how you got from that summation to the Θ complexity bound.

3. Prof. Sue Persmart in the CS Department at Portland Institute of Technology (motto: “Our CS Department is the PIT’s!”) likes to tell a story about the invention of chess.
- (a) According to legend, the game of chess was invented long ago in India by a certain sage. When he took the invention to his king, the king liked the game so much that he offered the inventor any reward he wanted. The inventor asked for some grain to be obtained as follows: Just one grain of wheat was to be placed on the first square of the chessboard, then two grains on the second square, four grains on the third square, eight grains on the fourth, etc., until all 64 squares had been filled.
- If it took 1 second to count each grain of wheat, how long would it take to count all the grains of wheat due to the sage?
- (b) Imagine that, instead of doubling the number of grains for each square of the chessboard, the inventor asked for adding two grains. Then (assuming again that it took 1 second to count each grain) how long would it take to count all the grains of wheat due to the sage?