

Theory of Computation
CS 378
Fall 2019

Problem Set 2

Due **AT THE BEGINNING OF CLASS** Thursday, October 3

- This homework covers material from pages 44-83 in Sipser and some additional topics that are not found in Sipser.
 - When using the Distinguishability Lemma to prove that a language is not regular, you must do the following: First, give an infinite set of strings S . Then, prove that for *any* two strings chosen from this set, those two strings are pairwise distinguishable. It does not suffice to show that *some* two strings in this set are pairwise distinguishable. (Do you understand why?)
 - *A general note:* When writing up your homework, please explain your arguments clearly and write neatly. Graders may not award credit to incomplete or illegible solutions. Clear communication *is* the point, on every assignment.
 - *A general note:* Neatly written (or typeset) solutions, with enough blank space on the page to allow graders to write comments before returning the papers, are greatly appreciated!
1. **Binary Addition is Regular!** Professor A. Tom Attah from the Massachusetts Institute of Typography is studying an interesting formal language. To begin, let our alphabet Σ be the set of all 3×1 binary vectors:

$$\Sigma = \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \dots, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$$

A correct addition of two binary numbers can be represented by a string in Σ^* . For example,

$$\begin{array}{rcccc} & 0 & 1 & 0 & 1 \\ + & 0 & 1 & 1 & 0 \\ \hline & 1 & 0 & 1 & 1 \end{array}$$

would be represented by the following string of four symbols from Σ .

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

The language $\mathcal{L}$ is the set of all strings in Σ^* representing correct additions. Show that Professor Attah's language is regular by constructing a DFA for this language. Explain briefly what each state of your DFA represents. (*Note:* Remember that a DFA scans its input tape from left to right but addition is performed from right to left. This is what makes this problem interesting! It's possible to solve this problem with a DFA with very few states—needlessly complicated DFA's may not receive full credit on this exercise.)

2. **DFAs and NFAs.** Let L be the language of strings w over the alphabet $\{0, 1\}$ such that w contains the substring $0ab0$ or $1ab1$ where $a, b \in \{0, 1\}$.
 - (a) Draw the state diagram of a DFA that accepts L .
 - (b) Draw the state diagram of a NFA that accepts L . Your NFA must use non-determinism and should have at most 9 states and 11 edges (an edge labeled with both 0 and 1 counts as a single edge).
3. **Another Closure Property of the Regular Languages.** Let L be a language. Then L^R , the *reversal* of L , is defined as $L^R = \{w^R | w \in L\}$. (Recall that w^R denotes the reversal of string w .) Prove that the regular languages are closed under reversal. That is, show that if L is a regular language then L^R is also regular. You may use drawings to help you explain your construction, but make sure that your accompanying explanation is clear.
4. **Yet Another Closure Property of the Regular Languages!** Show that if L is regular, then so is

$$HALF\ PALINDROME(L) = \{x | xx^R \in L\}.$$

Please explain your construction carefully!

5. **Nonregular Languages.** Prove that each of the languages below is not regular, using the Distinguishability Lemma. Your proofs must be clear, complete, and rigorous.
 - (a) $L = \{0^i 1^{2i} | i \geq 0\}$.
 - (b) $L = \{0^i 1^j 0^{i+j} | i, j \geq 0\}$.
 - (c) $L = \{ww^R | w \in \{a, b, c\}^*\}$.
6. **Short Answers.** In answering the following questions, you may use any results that we've seen in class. Each of these questions can be answered with a very short proof, counterexample, or small DFA or NFA.
 - (a) In class we showed that the union of two regular languages is always regular. Is the union of an infinite number of regular languages always regular? If yes prove it, if not give a counterexample.
 - (b) Is a subset of a regular language always regular? If yes prove it, if not give a counterexample.