

Theory of Computation
CS 378
Fall 2019

Problem Set 6

Due **AT THE BEGINNING OF CLASS** Tuesday, November 26

- Please read Sipser Ch. 3 and 4.
- *A general note:* When writing up your homework, please explain your arguments clearly and write neatly. Graders may not award credit to incomplete or illegible solutions. Clear communication *is* the point, on every assignment.

1. **TMwork!** Draw a transition diagram for a Turing machine recognizing the language below. Be sure to label the TM states with descriptive names.

- $\{a^i b^j \mid i < j\}$

2. **Space Warp Turing Machines!** NASA would like to send a Turing machine on a mission to a distant galaxy where there are holes in the space continuum. A *space warp* TM is a TM with a two-way infinite tape that operates under the following adverse conditions. At any moment during its computation, it may accidentally fall into a “space warp”, at which point the tape head is immediately transported to an arbitrary location on the two-way infinite tape. At the same time, its state is changed (involuntarily!) to a special state q_{warp} . Fortunately, NASA has determined that the number of space warps occurring during any computation is some fixed constant α , although no one has yet determined the value of α . As NASA’s Director of Theoretical Computer Science, your job is to show that any computation carried out by a regular TM can be carried out by a space warp TM. In other words, give a careful and complete (English) description of how a space warp TM can be programmed to recover from any interruption and continue its computation. Note that the machine may be interrupted while it is trying to recover from a previous interruption!
3. In each case below, determine whether the given set is countable or uncountable. As always, prove your answer. ($\mathbb{N}$ refers to the set of natural numbers.)
 - (a) The set of all finite subsets of $\mathbb{N}$. In addition, explain why the diagonal argument cannot be used to show that this set is uncountable.
 - (b) The set of all functions from $\mathbb{N}$ to $\{0, 1\}$.
 - (c) The set of all functions from $\{0, 1\}$ to $\mathbb{N}$.
 - (d) The set of all regular languages over $\{0, 1\}$.