

Theory of Computation
CS 378
Fall 2019

Problem Set 7

Due **BY THE END OF THE DAY** Friday, December 6

- *A general note:* When writing up your homework, please explain your arguments clearly and write neatly. Graders may not award credit to incomplete or illegible solutions. Clear communication *is* the point, on every assignment.

1. L_{union}

Consider the following language:

$$L_{\text{union}} = \{ \langle M_1, M_2, M_3 \rangle \mid M_1, M_2, M_3 \text{ are TMs and } \mathcal{L}(M_1) \cup \mathcal{L}(M_2) = \mathcal{L}(M_3) \}$$

Prove by *reduction* that this language is not recursive.

2. **Oracles!** You have been hired as the chief architect at International Turing Machines Corporation (ITM), a major manufacturer of theoretical computing machines. One day you receive a package in the mail postmarked from Delphi, Greece. The package contains a black box labeled “An Oracle for the Halting Problem.” The instructions explain that the oracle may be “connected” to any Turing Machine as follows. You connect the oracle to your Turing Machine by simply introducing three new states to your TM: q_{oracle} , q_{Yes} , and q_{No} . When your Turing machine enters state q_{oracle} with a machine-word pair $\langle M, w \rangle$ at the front of its input tape, the oracle will instantly cause your TM to enter state q_{Yes} if Turing machine M halts on w , and will cause your machine to enter state q_{No} otherwise.

You are **very excited** by your good fortune. Show that even the diagonal language, L_d , is decidable with this oracle.