

Analysis of Algorithms
CS 376, Fall 2020

Problem Set 4

Due **BY 11:59PM, Maine time** Thursday, October 22

Please submit this assignment in a file named `CS376_PS4_<YourName>.pdf`.

- From CLRS, please read Chapter 4 (4.1 is skippable).
- *A general note for CS376:* When writing up your homework, please write neatly and **explain your answers clearly**, giving all details needed to make your answers easy to understand. Clear communication *is* the point, on every assignment.

In general in CS376, unless explicitly specified otherwise, answers should be accompanied by explanations; as part of this, in general, when you present an algorithm, give an accompanying correctness argument (i.e., proof). Answers without explanations may not receive full credit. Please feel free to ask me any questions about explanations that might come up!

Exercises

1. CLRS 4.2-7 (page 83).
2. CLRS 4-3, parts a, c, and f (page 108).
3. Prof. Nigma at the Portland Institute of Technology (motto: “We don’t think about acronyms!”) hired you to analyze the algorithm given here in pseudocode (written by one of the Prof.’s Ph.D. students), but as usual, Prof. Nigma neglected to explain what the algorithm does.

$M(x, y) :$
if $y = 0$ return 0 else
return $M(2x, \lfloor y/2 \rfloor) + x \cdot (y \bmod 2)$

- (a) What does this algorithm do? Give a clean, concise specification for the algorithm, and then show the algorithm satisfies that specification; as usual, explaining how the algorithm works will be part of your argument. (You do not need to give examples as part of your answer, but you are welcome to do so if it would make your answer clearer.)
- (b) Express the running time of the algorithm in a recurrence *as a function of the length of y only*—i.e., treat x as if it were of fixed length for this analysis and analyze the asymptotic running time as the length of y changes.
- (c) What method for solving recurrences (of the four presented in class this semester) would you use to solve the recurrence from part 3b? What are your reasons for that? (You are welcome to solve the recurrence as well, just to find out the runtime of the algorithm, but that isn’t required for this exercise).

For this exercise, assume the following:

- x and y are positive integers
- checking if a number is equal to 0 is a constant-time operation
- the addition of two integers is a constant-time operation
- the operation of finding an integer mod 2 is a constant-time operation
- the operation of integer division by 2 (e.g., $\lfloor y/2 \rfloor$) is a constant-time operation

If you're not sure what makes any of these assumptions reasonable in this context, please ask me!