

1 Incentive compatible mechanism

A mechanism is a type of social choice function.

In particular, the function selects from a collection of known options, typically with the goal of satisfying some sort of a social welfare function.

In addition, the mechanism can specify payments made to / by each player.

Auctions:

I will imagine these as “sealed-bid,” where bids are made privately and then revealed. Typically, we assume whoever makes the highest bid gets the item. No changing bids afterwards.

Option 1: whoever bids the most gets the item and pays nothing. Clearly not incentive compatible.

Option 2: whoever pays the most pays their bid.

Two players A and B. A values the item at \$10. B values the item at \$8.

If A bid \$9, they would still win the item and only paid \$9.

Option 3: Use the second highest-bid.

This mechanism is incentive compatible!

That is, my dominant strategy is to bid my honest valuation of the item.

Imagine, my valuation is v and I bid b .

If I win by bidding b , let b' be the second highest bid, so my utility is $v - b'$.

- if $b > v$: If I instead bid v , do I still win? If $v > b'$, my utility is $v - b'$, so unchanged utility. If I don't win, then my utility having bid b was negative. By bidding v and not winning, I now have 0 utility, so I'm better off.
- if $v > b$: By bidding v , I have an even higher bid so I still win, so my utility stays the same. If I don't win by bidding b , my utility is currently 0.
- if $v < b$: By bidding v , I still wouldn't win, so still 0 utility.
- if $v > b$: By bidding v , I either still don't win in which case my utility remains 0, or I win, in which case I now have some positive utility, which is better than 0.

Some “nice” criteria I might want from my mechanism:

1. Incentive Compatibility
2. Players don't walk away with negative utility: this is called individual rationality
3. I probably don't want to pay players to use my mechanism

2 Vickrey-Clarke-Groves Mechanism

- Every player specifies their valuation over every option the mechanism chooses between
- The choice made by the mechanism maximizes “social welfare,” ie the sum of everyone’s reported valuations
- Each player pays the sum of 2 things:
 1. - (negative) the sum of everyone else’s value
 2. h_i this number only depends on everyone else’s valuations

What is player i’s utility under a VCG mechanism? Let’s say option “a” is chosen.

$$v_i(a) - (\text{what they pay}) = v_i(a) - h_i + \sum_{j \neq i} v_j(a) = \sum_j v_j(a) - h_i$$

So, by playing to their own advantage, everyone is trying to maximize global social welfare.

Let’s imagine an auction using the VCG mechanism with $h_i = 0$. Since the mechanism chooses the option that maximizes the sum of everyone’s values and the value of every option in which the player doesn’t win is 0, whoever bids the most gets the item. Let’s say I sort the players by reported value, i.e. $v_1 > v_2 > v_3 > \dots$ $p_1 = 0, p_2 = -v_1, p_3 = -v_1 - v_2, p_4 = -v_1 - v_2 - v_3$

Is this auction incentive compatible? Yes.

Everyone has non-negative utility.

We paid a lot.

Claim: Every VCG is incentive compatible.

Player i’s net utility: $\sum_j v_j(a) - h_i$ where a is the option chosen.

!a maximizes the sum! (according to VCG)

Imagine i instead reported v_i' and the option chosen was a' .

Difference in the utility: $\sum_j v_j(a) - \sum_j v_j(a') \geq 0$

Clarke-Pivot rule: setting $h_i = \max (\text{of all options } b) \sum_{j \neq i} v_j(b)$

Auction using Clarke-Pivot rule: Imagine we reorder the bids so that $v_1 > v_2 > v_3 > \dots$

Player 1 pays: h_1 - the sum of everyone else’s valuation of player 1 winning = max (of all options

b) $\sum_{j \neq 1} v_j(b) = v_2$

Player 2 pays: h_1 - the sum of everyone other than 2’s valuation of Player 1 winning = $v_1 - v_1 = 0$

Every player gets non-negative utility: Utility of player i = $\sum_j v_j(a) - h_i = \sum_j v_j(a) - \max (b) \sum_{j \neq i} v_j(b) \geq \sum_j v_j(a) - \sum_j v_j(b) \geq 0$ (if $v_j(b) \geq 0$)

$h_i - \sum_{j \neq i} v_j(a) = \max (\text{options } b) \sum_{j \neq i} v_j(b) - \sum_{j \neq i} v_j(a) \geq 0$.