

Theory of Computation
CS 378

Spring 2021

Problem Set 5

Due **AT THE BEGINNING OF CLASS** Tuesday, Apr. 27

- **Submission instructions:** Please submit this assignment to your SubmittedWork folder in a file named CS378_PS5_<userid>.pdf.
 - Please finish reading Sipser Ch. 2.1–2.3.
 - As always, answers to all exercises should be accompanied by explanations; brief explanations sometimes suffice, but all answers should be explained. Answers without explanations may not be awarded full credit.
 - *A general note:* When writing up your homework, please explain your arguments clearly and write neatly. Graders may not award credit to incomplete or illegible solutions. Clear communication *is* the point, on every assignment.
 - *A general note:* Neatly written (or typeset) solutions, with enough blank space on the page to allow graders to write comments before returning the papers, are greatly appreciated!
1. **Fun with Hans und Franz!** Use the Pumping Lemma for CFL's to show that the following language is not context-free:

$$\{a^n b^n c^m \mid n \leq m \leq 2n\}.$$

2. **Closure Properties of the CFL's.** Sipser 2.16.

3. **The Context-Free Languages are Not Closed Under Intersection!**

In this problem you will show that *unlike* the regular languages, the context-free languages are *not* closed under intersection. To show this you need only exhibit two context-free languages whose intersection is not context-free.

Give two context-free languages L_1 and L_2 such that $L_1 \cap L_2$ is not context-free. To show that L_1 and L_2 are context-free, give grammars or PDAs (or English descriptions of how the PDAs would work) for these two languages. You should use the pumping lemma for CFLs to show that $L_1 \cap L_2$ is not context-free. However, if $L_1 \cap L_2$ is already a language that we have shown in class to be not context-free, you can cite that proof without redoing it.

HINT: Sipser's exercise 2.2(a), on page 154, suggests an approach to this exercise. You are welcome to use that approach or a different one, whichever you prefer.

4. **FMP's (Funky Modified PDA's), part 1—Bottom Window PDA's!** Professor Graham Aire of the Portland Institute of Technology (motto: “We think our acronym is peachy!”) is studying *bottom window PDA's*—a kind of PDA that “can tell” at any time what symbol in the stack alphabet is at the bottom of the stack. That is, this kind of PDA has a window at the bottom of the stack that it can use to peek and see what's there. More formally, the transition function of such a machine would be

$$\delta : Q \times \Sigma_{\epsilon} \times \Gamma_{\epsilon} \times \Gamma_{\epsilon} \rightarrow \mathcal{P}(Q \times \Gamma_{\epsilon})$$

where the input comprises the state of the PDA, the symbol to read from the input tape (or ϵ if nothing is to be read and the machine will operate on the stack without reading the tape), the symbol to be popped from the stack (or ϵ if the machine ignores the top of the stack), and the symbol on the bottom of the stack (ϵ if the stack is empty), and the output is as for a standard PDA.

Prof. Aire's intuition is that a bottom window PDA would be more powerful than a standard PDA, because it can do more than a standard PDA—it can look at the bottom of the stack (though it can only peek there, not push or pop there), which a standard PDA cannot do.

We can understand that intuition, but ... in fact, a bottom window PDA is *not* more powerful than a standard PDA! Prove this by giving a full explanation of how, in general, any bottom window PDA could be simulated by some standard PDA.

Optional: More Closure, and More FMP's!

The exercises below are not to be turned in—they're just exercises that your Prof. thought might be of interest! You are welcome to work on them and talk with your Prof. or TA about them, but they will not be graded.

- **The Context-Free Languages are Not Closed Under Complementation!**

On the previous problem set, the optional exercise (“**And Now For Something Truly Funky!**”) established that the language

$$(a + b)^* - \{(a^i b^i)^i \mid i \geq 1\}$$

is context-free.

Now, show that the *complement* of this language, $\{(a^i b^i)^i \mid i \geq 1\}$, is NOT context-free using the Pumping Lemma for CFL's. Note that this result demonstrates that the context-free languages are not closed under complementation!

- **Middle Window PDA's!** Prof. Aire of the PIT is also interested in Middle Window PDA's!

Consider a PDA that “can tell” at any time what symbol is in the middle of its stack. That is, the PDA has a window that allows the PDA to see the middle symbol. If the PDA has an odd number of symbols on the stack, the middle is well-defined. If the PDA has an even number of symbols on the stack, we'll define “middle” to mean the symbol just above the midpoint (the “higher of the two possible middles”). Note

that the middle window “floats” up and down as the number of elements on the stack changes.

Is such a machine more powerful than a regular PDA? If so, demonstrate a language that is not context free but is accepted by a middle window PDA (prove that this language is not context free or use a language that we’ve already shown to be a non-CFL) and explain how it is accepted.

If not, explain how a regular PDA can simulate a middle window PDA.